Efficiently Wasting Fuel

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In a previous article (May/June 98 pp. 14, 15) we discussed range. Specifically, we found that the velocity for maximum range occurred at the velocity for $V_{L/D_{\max}}$, i.e., the maximum glide velocity. We also found that the maximum range was given by the so called Breguet equation

$$R = 326 \frac{L}{D} \frac{\eta}{c} \ln \frac{W_0}{W_1}$$

where R is the range in nautical miles, L/D is the lift to drag ratio, η is the propeller efficiency, c is the specific fuel consumption in lb/bhp-hr, W_0 is the weight at the beginning of cruise, W_1 is the weight at the end of cruise and 326 is a conversion factor so that the result is given in nautical miles. ln means the natural logarithm. You can find the specific fuel consumption in your engine operating handbook. The Breguet equation shows that to maximize the range, i.e., milage (nm/gallon of fuel), we must fly at the velocity for $V_{L/D_{\max}}$, maximize the propeller efficiency by selecting an appropriate propeller RPM and minimize the specific fuel consumption by properly leaning the engine.

Relative to other transportation systems, airplanes are designed to go *fast*. In fact, airplanes are unique among transportation systems because they are designed to go fast *economically*. For example, with our Bonanza it is easy to visit two of our five grandchildren, who live 325 nautical miles away, *for lunch* and at lower out-of-pocket cost than if we took the three days to drive the 425 statute miles each way for the same trip. On another scale, if you are a businessman living in London, England, you can hop on the early morning Concorde flight for a morning meeting in New York and be back in London that evening. Time has value.

The equivalent airspeed, EAS, for $V_{L/D_{\max}}$ at gross weight for an E33A/F33A is 121/122 mph, or 105/106 kts. At sea level, of the 285 BHP available the power required to achieve this EAS is only about 90 horsepower, i.e., about 32% of the available power. Furthermore, the manifold pressure and RPM to achieve this horsepower is outside the recommended continuous cruise settings in the POH. In addition, at this low velocity and propeller RPM the propeller efficiency is not optimum and the specific fuel consumption is actually higher than at higher propeller RPM. Besides, if we wanted to cruise at 105 kts we would not fly a Bonanza.

The chief reason that a fundamental mismatch between the available power installed in an aircraft and that required for efficient cruise is that an aircraft must be able to climb. Climbing requires excess power over that required to maintain level flight. The real question, then, is how do we use this excess power to fly fast economically?

In the early 1980s B. H. Carson, a long time colleague, answered this question.† He noted that the power required to maintain level flight, P_{req}, is proportional to the rate at which energy (fuel), E, is expended per unit time, t. He then related the energy expended per unit distance traveled, s, to the power required divided by the velocity. The power required divided by the velocity, V, is just the drag, D, which can be related to the weight, W, times the lift to drag ratio, L/D. Mathematically, we write this as

$$\frac{dE}{ds} = \left(\frac{dE}{dt}\right)\left(\frac{dt}{ds}\right) = \frac{P_{\text{req}}}{V} = D = \frac{W}{L/D} = W\frac{D}{L}$$

From this we again see that, for a given weight, the range, R, is maximized when the lift to drag ratio is maximized because the expenditure of fuel per unit distance is minimized. The designer's dilemma is to maximize the lift to drag ratio while efficiently using the excess power required for climb to fly fast

†Carson, B. H. Fuel Efficiency of Small Aircraft, *AIAA J. of Aircraft*, Vol. 19, No. 6, pp. 473–479, June 1982.

in cruise. Returning to our familiar power required equation and dividing by the velocity clarifies the dilemma

$$\frac{P_r}{V} = D = \frac{D}{L}W = \underbrace{\frac{\sigma\rho_{\rm SL}}{2} f V^2}_{\text{parasite}} + \underbrace{\frac{2}{\sigma\rho_{\rm SL}}\frac{1}{\pi e}\left(\frac{W}{b}\right)^2 \frac{1}{V^2}}_{\text{effective induced}}$$

Dividing by the weight, W yields

$$\frac{D}{L} = \frac{\sigma\rho_{\rm SL}}{2}\frac{f}{W}V^2 + \frac{2}{\sigma\rho_{\rm SL}}\frac{1}{\pi e}\frac{W}{b^2}\frac{1}{V^2} = AV^2 + \frac{B}{V^2}$$

where

$$A = \frac{\sigma\rho_{\rm SL}}{2}\frac{f}{W}; \qquad B = \frac{2}{\sigma\rho_{\rm SL}}\frac{1}{\pi e}\frac{W}{b^2}$$

Now, to maximize the lift to drag ratio, L/D, the drag to lift ratio, D/L, needs to be minimized. From the above equation this is accomplished by making *both* A *and* B as small as possible. Well, that is not going to happen. Here is why. Suppose the weight, W, is decreased to make B smaller, but then A becomes larger. Suppose the altitude, σ, is increased to make A smaller, but then B becomes larger because σ, the density ratio, becomes smaller. Ah ha, you say the wing span, b, only appears in B and we can make B smaller by increasing the span, i.e., the aspect ratio. But, if the span is increased, then the area of the wing skin is increased which increases the parasite (skin friction) drag, which in turn increases A. The weight of the wing also increases to support the longer span, hence B increases. Finally, e, the Oswald or airplane efficiency factor, stubbornly sticks to a range from about 0.6 to 0.8, so it is not much help.

As a result, because the designer needs to meet a rate-of-climb requirement, or requires additional thrust or power available at sea level to meet a high altitude cruise requirement and because, as we saw above, the velocity for maximum lift to drag ratio cannot be increased to meet the high speed cruise requirement, aircraft cruise at nonoptimum high speeds.

Carson asked the question "What is the unit cost in increased fuel consumption for each unit increase in speed?" His results showed that the best rate of return for increased fuel consumption as a result of increased cruise speed was

$$V_{\rm cruise} = (3)^{1/4} V_{L/D\,\rm max} = 1.32 V_{L/D\,\rm max}$$

while the increase in fuel consumption was $2/\sqrt{3} = 1.16$, i.e., a 32% increase in cruise TAS for just a 16% increase in fuel consumption. The increase in cruise TAS resulted in a 52% increase in power required and a 24% decrease in the flight time. From this one can also conclude that the value of a 24% decrease in flight time is worth the cost of an additional 16% for fuel. $V_{\rm cruise}$ has become known as the Carson cruise, e.g., CAFE (Comparative Aircraft Flight Efficiency) calculates the Carson cruise speed when evaluating an aircraft.

Table 1 True Airspeeds (kts)

Altitude feet	$V_{L/D\,\rm max}$	$V_{\rm cruise}$	75% BHP	65% BHP	55% BHP
0	105	139	163	154	143
2000	108	143	166	157	146
4000	112	147	169	159	148
6000	115	152	172	162	149
8000	119	156	171*	164*	151
10000	122	161	169*	161*	150*
12000	126	166	166*	158*	146*
14000	131	172	163*	158*	141*

From a practical viewpoint, how do we apply the Carson cruise results to every day operation? First, as shown in Table 1, remember that the true airspeed for $L/D_{\rm max}$ increases with increasing altitude. Hence, the Carson cruise true airspeed also increases with increasing altitude. Also shown in Table 1 are the true airspeeds for 75% (2500 RPM), 65% (2300 RPM) and 55% (2100 RPM) BHP (brake horsepower) taken from the POH for a 285 BHP model E33A Bonanza for a weight of 3100 lbs. The asterisks indicate operation at full throttle and the indicated fixed RPM, i.e., at less than the indicated percentage of BHP. Comparing the $V_{\rm cruise}$ true airspeeds and those for 75% BHP, we see that at 12,000 feet the true airspeeds are equal at 166 kts. Similarly, for 65% BHP the true airspeeds are equal at 10,000 feet pressure altitude at 161 kts. At 55% BHP, note that the true airspeeds do not match at any altitude. Thus, at 55% BHP (2100 RPM) flight at the Carson cruise true airspeed is not possible at any altitude. This is not surprising. Note that for both 75% BHP (2500 RPM) and 65% BHP (2300 RPM) operation is at full throttle and a fixed RPM, hence at less than the indicated percentage of brake horsepower. Carefully checking the altitude performance curves from the POH shows that operation at both 2500 RPM and 2300 RPM at full throttle at the required altitudes yields about 59% BHP.

Normally aspirated reciprocating engine propeller combinations have a critical altitude at which they can maintain a specified percentage of sea level BHP. This critical altitude is sometimes called the knee of the altitude-true airspeed curve. For a given percentage of sea level BHP, the knee represents the most efficient operating condition, i.e., the highest TAS for a given percentage of sea level BHP and hence fuel expended. As an example, for a model E33A at a weight of 3100 pounds at 65% BHP on a standard day the knee of the curve occurs at approximately 7600 feet pressure altitude and yields a TAS of 164 kts.

By comparing the Carson cruise TAS, V_{cruise}, and the knee velocity, V_{knee}, as a function of percent BHP using the two curves in the lower plot of Figure 1, we see that the two curves cross at approximately 58.6% BHP, yielding a TAS of approximately 157 kts (180 mph). For 58.6% BHP, the knee in the altitude vs TAS curve occurs at a pressure altitude of approximately 8300 feet, as shown by the upper plot in Figure 1. For this power the approximate fuel flow is 12.5 gph, which yields a milage of approximately 12.6 nautical miles per gallon (14.5 statute miles per gallon). Compared to an SUV that gets about the same mileage, weighs about the same but moves at 40% of the speed and typically has to cover a 20% greater distance to get from point A to point B, that's not too shabby.

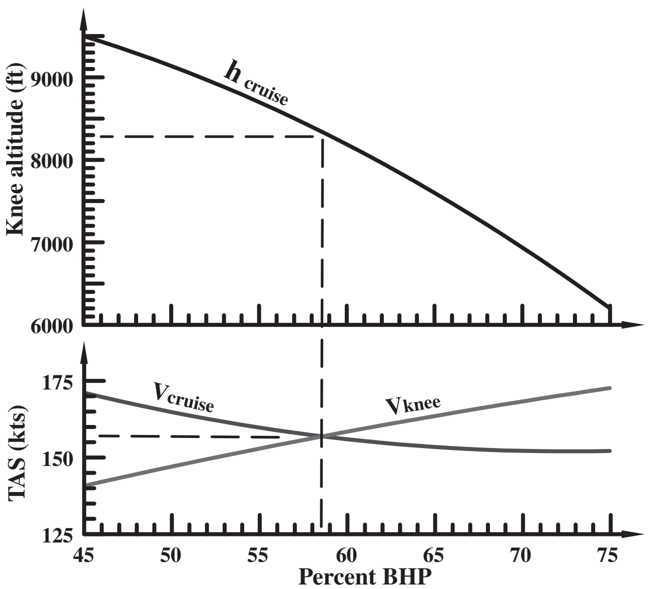

Figure 1. V_{cruise} and h_{cruise} vs percent brake horsepower available.

Dave Rogers' previous articles are available through a link at his website at *www.nar-associates.com*.